# GRAPHICS CALCULATORS IN UNDERGRADUATE MATHEMATICS TEACHING

**Sandra Britton**
School of Mathematics and Statistics
University of Sydney

## Abstract

This paper will report on an on-going project, funded by CAUT (Committee for the Advancement of University Teaching), involving the use of TI-85 graphics calculators in the teaching of a first year mathematics class at Sydney University. The calculator was introduced to the 1993 class as a demonstration tool during lectures. New tutorial material has been written for the current class, and students are using the calculators in tutorials. This paper will discuss findings from the initial stages of the project.

## The project

The aim of this project is to integrate the use of a graphics calculator into the teaching of a large first year mathematics class at Sydney University, and to evaluate the effect this has on student learning. In 1993 the calculators were used for demonstration purposes during lectures, and this year we have extended their use by having the students use them in tutorials. We have, therefore, developed new tutorial material (based on the existing syllabus). Our aim was to produce material which would encourage the students to use the calculator as an exploratory tool, and to enable them to solve practical, rather than contrived, problems.

The rationale for the project was the belief that the use of graphics calculators would have a strong positive impact on student learning. Various studies in the United States have reported such an impact at a pre-calculus level. (Demana and Waits, 1989, 1992; Shumway, 1990; Ruthven, 1990; Dion, 1990). Similarly, Boers and Jones report that "the introduction of the graphics calculator was associated with a dramatic improvement in performance" of certain groups of students in their study conducted at the Swinburne Institute of Technology in 1991.

The methodology employed to evaluate the effect on student learning includes student surveys at various times throughout the year, analysis of student performance on class tests and in examinations, and trials with a small group of students. In addition, the tutors meet weekly, and informal discussion relating to observations made by the tutors during tutorials is recorded.

## The class

The class at which the project is directed is a first year class of approximately 350 students, called Mathematics I (Life Sciences). This is a service course in calculus, linear algebra and statistics for students in the life sciences. The assumed knowledge for the course is 2 Unit Mathematics, although the class includes a large proportion of students with 3 or 4 Unit Mathematics, as well as a small number with less than the assumed knowledge. (Students in the latter category are advised to take a bridging course in January/February before attempting the course.) The class contains mostly Science students, with some students from the faculties of Arts, Economics and Education. Approximately two-thirds of the class is female. Maths I (Life Sciences) is a terminating course, and for most students it is their last formal mathematics course.

In a survey conducted in the second week of first semester this year, 40% of the class professed to anxiety about mathematics, while around 60% claimed that they were studying mathematics only because it is a requirement of their degree. (Interestingly, however, only 25% claimed that mathematics was their least favourite subject.) There is strong evidence to suggest that students who feel threatened by, or anxious about, mathematics gain particular benefit from using a graphics calculator.

## The calculator

The calculator we are using is Texas Instuments' TI-85. For those unfamiliar with the TI-85, it is a highly sophisticated machine, about the same size as an ordinary scientific calculator, with the ability to display graphs of functions (along with a host of other capabilities). It can, of course, be used as a normal scientific calculator. It is operated by selecting menus, and sub-menus. For example, a few of the menus are GRAPH, CALC (for calculus), STAT (statistics), SOLVER, MATRIX. In the GRAPH menu, one can enter a function and explore its graph by tracing along the curve, zooming in or out, and looking for maximum or minimum values, or points of inflection, with the press of an appropriate button. The current price of this calculator is around $230. We have not prescribed the purchase of a calculator as essential for the course. The School of Mathematics and Statistics has purchased 30 TI-85's, and the students use these in tutorials.

While the use of graphics calculators is not currently widespread in Australia, it seems that in the United States such calculators are rapidly becoming standard equipment for typical science undergraduates. Indeed, graphics calculators are required for many "reformed calculus" courses, and from 1995 will be required for the AP Calculus exam. I believe that a similar situation will develop here, and an important aspect of this project is that the students, many of whom are prospective high school teachers, will be familiar with this

technology. Equally important is the consideration that learning to deal with new technology should be part of any science student's education.

### Use of the calculator in lectures

At the beginning of 1993 the School of Mathematics and Statistics purchased an overhead projection device (the ViewScreen) for the TI-85, and this was used during lectures to the class throughout 1993. In association with the ViewScreen the calculator offers enormous scope for fast, effective illustrations of certain concepts. (What's more, it draws graphs much more accurately than most of us are capable of drawing on a blackboard!) The students seemed to enjoy the demonstrations given on the ViewScreen, and gasps of wonder were sometimes heard as the calculator performed certain operations. At the end of the year, when the students were surveyed they were asked to say what the best and worst aspects of the course were. One students' response to the best aspect question was: "The super-dooper calculator!"

### The new material

In this section I would like to give a few illustrations of the types of exercises we have asked the students to do, using the calculator. The first topic we teach in this course is linearisation, using log-log and semi-log transformations. The types of questions we ask of the students are particularly well-suited to the use of the TI-85. Here is one example:

> Use the TI-85 to find a transformation which linearises the following data, and hence determine whether $y$ is related to $x$ by the exponential law $y = Ae^{bx}$, or by the allometric function $y = Ax^b$.
>
> | | | | | | | | |
> |---|---|---|---|---|---|---|---|
> | $X$: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
> | $Y$: | 11 | 4.5 | 1.8 | 0.7 | 0.3 | 0.1 | 0.05 |
>
> Find the values of $A$ and $b$, and check that your equation fits the data by drawing (on the TI-85) the graph of your function over a scatter plot of the data.

(On the TI-85 one can store data as a list, take logs of the list, draw a scatter plot of one list against another, and perform a linear regression.)
The following example is a tutorial question on modifications of sinusoidal functions:

> Use the TI-85 to draw (on the same screen) the graphs of (a) $y = \sin x$ and (b) $y = 2 + \sin(0.75x)$. State the period and amplitude of the function in (b), and describe (in words) the modification of graph (a) which produces graph (b).

(The standard viewing rectangle on the TI-85 is $-10 \leq x \leq 10$, $-10 \leq y \leq 10$. There is a ZTRIG option in the ZOOM menu which sets the range so that it is suitable for viewing the graph of $y = \sin x$.)
We have also been able to include exercises which allow students to see graphs of functions which they would not be able to draw for themselves. For example:

Find any maximum and minimum values of the function $y = e^{(x-4)} - 5x^2 + 8$.

(The standard viewing rectangle will not display a useful graph in this example. The exercise is not merely one of button-pushing, since one needs to have some idea of what to expect before a suitable range is chosen.)
As a final example, here is a question which we would not have been able to ask previously:

Evaluate $\sum_{n=1}^{1000} \frac{1}{n^2}$; $\sum_{n=1}^{2000} \frac{1}{n^2}$; $\sum_{n=1}^{2500} \frac{1}{n^2}$. Do you think that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ exists?

There are very many capabilities of the calculator which students will not use during this course. The TI-85 manual consists of more than 200 pages, and expertise in the use of the TI-85 requires quite some time. We have written instructions – "hints for using the calculator" – to accompany each of the tutorial exercises. These "hints" give a brief explanation of the functions of the calculator which are useful for the exercise, and describe relevant sequences of keystrokes. The students are not, of course, expected to work through these entirely on their own. Their tutor will always explain any aspect of the calculator which they have not previously seen.

### Preliminary trialling of the material

By early February this year we had prepared a "mini-manual" for the calculator, an introductory tutorial, and several sets of tutorial exercises and hints. We then employed a group of five undergraduates to work through the material as they would in a tutorial. (By chance, rather than design, these five were all female. Two had studied Maths I (Life Sciences) in 1993, one had studied Mathematics I in 1993 and the other two had just completed the HSC.) This was an extremely useful exercise and provided valuable feed-back on the efficacy of the hints, insights into the way in which students might use the calculators and an indication of problems they might encounter. We were encouraged by the fact that all five of these students seemed to enjoy using the calculator, and learnt to use it efficiently with reasonable ease. In their responses to a questionnaire at the end of the sessions all five claimed that they would recommend the TI-85 to other students, all five found the calculator interesting to use and four stated that the TI-85 had helped them to understand some of

the mathematics in their previous courses. One student wrote: "Introducing the TI-85 is a very positive step towards a greater understanding of maths for students". (These sessions were conducted over five days, for three hours each day, with myself and one other person acting as tutors. We were aware, of course, that things could be very different when we had tutorials of 25 - 30 students, with one tutor.)

### Results of a survey carried out in week 2 of semester 1

In the first week of semester students were meant to attend a tutorial at which they worked through a set of exercises designed to familiarise them with a limited number of functions of the TI-85. (Unfortunately, students had not received the correct information on their timetables, and so many students missed this introductory tutorial.) In the second week of semester we distributed a questionnaire which we asked the students to fill in and return. We received 148 replies. The survey included questions relating to general attitude to mathematics, previous experience with technology, and ways in which mathematics is "done". We plan to distribute a survey containing similar questions before the end of this semester. I am particularly interested in seeing whether the use of the calculator causes any significant changes to the way in which they "do" mathematics.

The survey also included an open-ended question which asked for first impressions of the TI-85. Of the 148 responses, 104 included comments on the TI-85. (44 students either did not answer the question or had not attended a tutorial.) Of these 104 comments, we classified 42 as favourable, 36 as unfavourable and 26 as ambiguous or non-committal. The unfavourable comments included words such as "confusing" and "complicated", while the favourable ones mentioned "interesting", "fun" and "Wow!!". Several comments were along the lines: "It will take time to learn to use it, but I can see that it could be very useful". While no-one said what they thought it would be useful *for*, my guess is that they thought it would be useful as a tool to carry out certain tasks, rather than as an aid to conceptual understanding.

### Results of a class test

Based on the belief that any change in teaching methods should be accompanied by a change in assessment methods, we decided to administer three class tests this semester. The tests are to count, collectively, for 10% of the assessment for the calculus component of the course. A further reason for this decision was the feeling that students would not take the calculators seriously unless we assessed their use somehow. Since the students are not required to own a calculator we clearly could not set exam questions requiring calculator use. In week 5 the first test was given, during tutorials. We set a question for each class which involved using

the TI-85 in exactly the same way they had been using it over the previous few weeks. Our initial reaction (before marking the test) was that the students were not doing well. We allowed them to work on the test in pairs, so that there was plenty of conversation to listen to while they were doing the test. Many students seemed to have forgotten how to use the calculator, and many were unable to complete the task in the 10 minutes we allowed them. However, after marking the test (out of 5) we found that, of 289 students, only 5 scored 0, while 60 scored 5, and 205 scored 3 or more.

### Perceived impact of the calculator

After seven weeks, the students seem to have accepted the TI-85 as part of this course. Efficient use of the TI-85 does take time and practice and, as we had expected, there was some initial frustration on the part of some students. In retrospect, I think that we should have chosen a different topic to work with as the introduction to the TI-85. The linearisation of data, and finding the regression line, involved moving between three different menus, whereas one can explore functions without moving out of the GRAPH menu.
Over the past two weeks many of the students have enjoyed graphing functions and their derivatives, and hopefully the use of the calculator has taught them something about the relationship between a function and its derivative. My overall impression is that the majority now feel reasonably comfortable using the calculator. I have been pleased to observe several students using the calculator to check their answers to assignment questions. Assignment questions are regular pen and paper type questions, not requiring calculator use. One question, for example, gave a sketch of a sine curve, and asked for a formula describing the curve. Two students, working together, had come up with a formula which they then checked by typing their equation into the TI-85 and sketching its graph. They then found that the graph on the TI-85 was not the one they had been given, so they tried again. For students who will be *users* of mathematics in their future studies and careers, this is precisely the sort of use of this technology that I would like to encourage.

### Conclusions

From an educator's point of view, I think that we would all agree that the advantage of using this type of technology lies in its potential for promoting understanding of mathematical concepts. The idea is that presenting a mathematical problem algebraically, numerically and graphically, aids understanding. It is an idea which has great currency amongst mathematics educators. It was certainly one of the major reasons for our project. We want our students to *understand* things, not just be able to "do" them. However, I do not believe that most students place the same premium on understanding. Most students, I believe, are content

with being able to "do" things – in particular, they want to be able to "do" the types of things they will be asked in an exam. Indeed, many students equate understanding with the ability to be able to perform certain tasks. (How many times have you heard a student say "I don't understand how to do this"?) They are therefore bound to see a graphics calculator as a useful tool which will allow them to "do" more mathematics, more easily, and they are unlikely to use such a tool with the specific purpose of increasing understanding.

While this project is still in the early stages, our findings so far are consistent with these views. I believe that the use of technology does have a positive impact on student learning, but that it would be fanciful to expect dramatic increases in conceptual understanding. (After all, understanding is *hard*. If we want more understanding, then we have to teach a lot less material.) Nevertheless, I believe that a certain amount of understanding seeps in while students are using graphics calculators. With the added benefits of increasing confidence and decreasing anxiety for many students, I can see no good reason for *not* using these calculators. We just shouldn't expect miracles.

## References

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